EgoMUIL: Enhancing Spatio-Temporal User Identity Linkage in Location-Based Social Networks With Ego-Mo Hypergraph

Haojun Huang, Fengxiang Ding, Hao Yin, *Member, IEEE*, Gaoyang Liu, *Member, IEEE*, Chen Wang, *Senior Member, IEEE*, and Dapeng Oliver Wu, *Fellow, IEEE*

***Abstract*—Users tend to own multiple accounts on different location-based social network (LBSN) platforms, and they typically engage with diverse social circles on each platform within the same locations. Consequently, linking these accounts across separate networks becomes essential, playing a critical role in information fusion. Previous works accomplishing user identity linkage (UIL) utilize individual mobility records, which are significantly affected by the issue of data scarcity. In this paper, we propose EgoMUIL, a heterogeneous graph embedding approach specifically devised for information propagation, aiming to alleviate the scarcity problem to some extent. Considering that follow relations of respective networks also hold great significance for the UIL task, we are inspired to enrich individual limited mobility records through follow relations. Our preliminary research reveals that direct common follow relations are quite insufficient. Since the followers with the same spatio-temporal mode tend to have social connections, we first mine closely-related users for each user through topology and locality similarity, generating respective cross-domain ego-networks. Subsequently, we construct a heterogeneous ego-mo hypergraph consisting of mobility and ego-networks. We propose a novel graph convolutional network (GCN)-based approach to learn user representations, which enables the aggregation of information from surrounding nodes, incorporating topological similarities, stay locality similarities, and co-occurrence frequencies. The resulting embeddings provide comprehensive representations of users and locations, capturing their characteristics and relationships across platforms, which further facilitates the UIL task. Our experimental results on real-world check-in datasets from Foursquare and Twitter demonstrate that EgoMUIL outperforms the state-of-the-art methods on the UIL task. Notably, EgoMUIL exhibits superior performance in scenarios involving limited check-in records and follow relations.**

***Index Terms*—User identity linkage, graph learning, topology similarity, link prediction, LBSN.**

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Haojun Huang is with the Hubei Key Laboratory of Smart Internet Technology, School of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan 430074, China, and also with the Department of Computer Science, University of Exeter, EX44QF Exeter, U.K. (e-mail: hjhuang@hust.edu.cn).

Fengxiang Ding and Chen Wang are with the Hubei Key Laboratory of Smart Internet Technology, School of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: dingfx@hust.edu.cn; chenwang@hust.edu.cn).

Gaoyang Liu is with the Hubei Key Laboratory of Smart Internet Technology, School of Electronic Information and Communications, Huazhong University of Science and Technology, Wuhan 430074, China, and also with the School of Computing Science, Simon Fraser University, Burnaby, BC V5A 1S6, Canada (e-mail: liugaoyang@hust.edu.cn).

Hao Yin is with the Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing 100084, China (e-mail: h-yin@mail.tsinghua.edu.cn).

Dapeng Oliver Wu is with the Department of Computer Science, City University of Hong Kong, Kowloon, Hong Kong (e-mail: dpwu@ieee.org).

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I. Introduction

LOCATION-BASED Social Networks (LBSNs) such as Twitter and Foursquare provide their respective users convenient platforms to generate and share their spatio-temporal records registered at point-of-interests (POIs) [1], [2], [3]. Analyzing a user's activity solely within a single LBSN platform may fall short in providing a comprehensive understanding of their activity characteristics. However, by incorporating spatio-temporal information, which encompasses both the spatial (location-based, e.g., check-ins) and temporal (time-based) aspects of social interactions or activities, we can gain more valuable insights into the complete mobility pattern of the user. One of the most representative techniques is the spatio-temporal user identity linkage (UIL), which enables data scientists and service providers to unlock the latent features that they cannot derive by mining user mobility on separate single platform, and is useful in various applications, such as user behavior prediction and cross-domain recommendation [4], [5], [6], [7].

Several approaches have been presented to tackle the problem of spatio-temporal UIL in LBSNs. A common practice is to align the user's trajectory data directly [8], [9], or transform nodes into embedding vectors where identities and "encountering" events are represented as vertices and edges respectively such that linked identities are closer in the embedding space [10], [11], [12]. For instance, Basik et al. [13] develop a summary representation for spatio-temporal records and align entities capturing the closeness in time and location. Recently, Feng et al. [4] introduce a multi-modal embedding network to extract representative features of individual's locations and trajectories.

However, existing efforts often suffer in the scenario where the generated check-in records per user are quite scarce, which means there are not enough spatio-temporal data to guarantee the quality of UIL. We analyze a set of widely used data provided by [14] to illustrate the deficiency of the data size per user and validate the impact on several typical UIL methods like DPLink [4], SIMP [15], POIS [16] and GKR-KDE [8] (detailed in Section II). It turns out that data scarcity does exist in commonly used datasets where users who generate less than five records account for almost 80% in Foursquare and Twitter platforms. Not surprisingly, these algorithms encounter obstacle when dealing with users with scarce check-in records. In particular, more than half of the users with less than five records could not be matched correctly. As a result, the practical implementation of these algorithms has been significantly hindered.

To address the data scarcity issue, in this paper, we present EgoMUIL (Ego-network and Mobility joint learning for User Identity Linkage), which can perform well even in the absence of enough aligned check-in records. The underlying assumption is that users with limited check-in records can be more accurately matched when considering their close friends due to their established social connections and corresponding physical locations in the real world. Consequently, our approach aims to incorporate both cross-domain ego-network factors and spatio-temporal characteristics in order to better capture the uniqueness of cross-domain users. These factors will be integrated into a dynamic hypergraph embedding space to facilitate the final UIL matching process.

Although the idea sounds straightforward, there still remain several technical challenges. First, the available follow relationships are often quite limited. Our research indicates that more than half of users follow fewer than 5 accounts. Second, the accounts that a user follows on different platforms can vary significantly due to specific purposes. For example, individuals may use Facebook primarily for communication with friends and family, while focusing on peers and celebrities on Instagram. Consequently, there is often little overlap between the accounts followed on different platforms. Recognizing this, we observe that social relationships can be flexible and extend beyond simply follow relationships on social networks. Drawing on earlier research demonstrating the potency of spatio-temporal information as a predictor of social connections [17], our work is inspired to explore the matching of users' follow relations across various social networks. Specifically, individuals observed together frequently at the same place and time slot are most probably socially related. Therefore, to address the issue of limited following accounts, we incorporate spatio-temporal associations as additional clues to match one user's social neworks and expand his relationships. Additionally, we propose a novel method that combines topology structure and locality area similarity to constrain and identify closely-related users across different networks.

Our major contributions are summarized as follows.

- We identify the low overlap phenomenon in cross-domain relation mining through empirical data analysis, which confirms the inadequacy of existing solutions in addressing the spatio-temporal UIL problem. To uncover potential social relations, we propose a novel method to establish the cross-domain ego-networks for users.
- We design an EgoMUIL framework to cope with the data scarcity issue in UIL. Our framework leverages the spatio-temporal properties of cross-domain ego-networks, wherein an ego-mo hypergraph is formed and a graph convolutional network (GCN)-based embedding mechanism is developed to learn user representations.
- We conduct extensive experiments using real-world check-in datasets from Foursquare and Twitter, and the results show that our EgoMUIL outperforms the state-of-the-arts on the UIL task. Notably, EgoMUIL exhibits superior performance in scenarios involving limited check-in records and follow relations.

TABLE I
DESCRIPTION OF DATASETS

Domain	Users	Edges	Check-Ins	Date Range
Foursquare	2,970	42,390	44,915	2008.10-2012.11
Twitter	3,518	96,579	516,787	2010.01-2012.11

The remainder of this paper is organized as follows. We present the motivation after empirical data analysis in Section II. We describe the detail design of EgoMUIL in Section III, followed by the performance evaluation in Section IV. Section V reviews some related works, and finally Section VI concludes the paper.

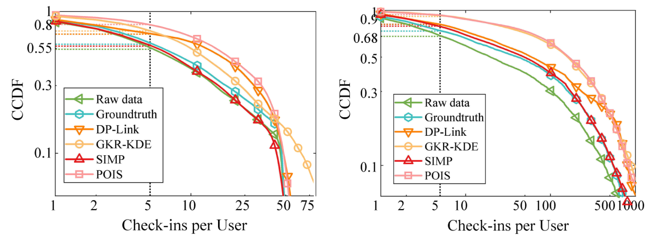
II. Empirical Data Analysis

A. Impact of Data Scarcity

We first analyze the available check-in data on a widely used Foursquare-Twitter dataset pair provided by [14] (data statistics are described in Table I) by implementing four state-of-the-art spatio-temporal UIL algorithms, including GKR-KDE [8], DPLink [4], SIMP [15], and POIS [16]. We plot the complementary cumulative distribution function (CCDF) curves of the number of check-in records per user on the original raw data (including the cross domain linked user pairs and the isolated accounts), the ground truth (i.e., the cross domain linked user pairs), and the linked user pairs returned by the above four algorithms, respectively (c.f. Fig. 1).

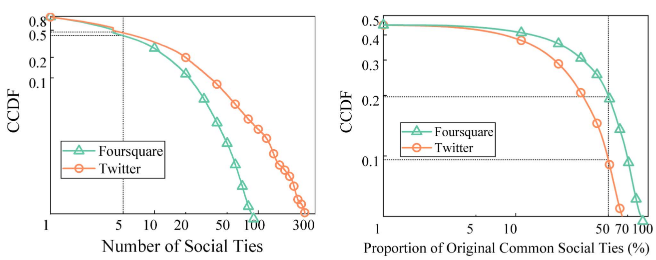
The analysis of the Foursquare dataset reveals that a significant portion of the raw data is covered by users with only a small number of check-in records; approximately 45% of users have less than five records. Not surprisingly, the scarcity of records leads to varying degrees of performance degradation for these UIL algorithms, with POIS being the most affected. Specifically, in POIS, the identified users with less than five records account for only about 20% compared to the 45% found in the raw data. This gap represents the users who failed to be correctly identified.

A similar trend is observed in the Twitter dataset, where all four algorithms underperform when facing scarce check-in records. The substantial performance degradation can be largely



(a) Foursquare (b) Twitter

Fig. 1. Complementary cumulative distribution function (CCDF) of the number of check-in records per user. The ground truth represents the data of users truly matched from the original raw data. We report the CCDF for linked user-pairs generated by GKR-KDE [8], DPLink [4], SIMP [15], and POIS [16] on the raw dataset.



(a) Number of social ties. (b) Proportion of cross domain common social ties.

Fig. 2. CCDF of the number of social ties and the proportion of common social ties in Foursquare and Twitter datasets.

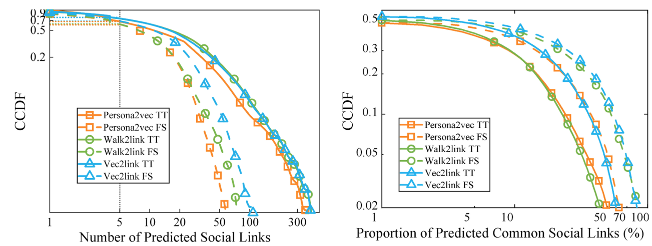
attributed to the insufficient user portrait created by the spatio-temporal features of the scarce check-in records, resulting in a higher possibility of identity mismatch.

As the scarcity of check-in records is an inherent challenge in practice, we are motivated to explore additional features that can enhance the UIL performance. These additional features may compensate for the limitations imposed by the sparse nature of check-in records and improve the accuracy of the UIL task.

B. Impact of Social Ties

Motivated by the network structure, which is widely used to characterize a user's social connections through follower-followee relations (also known as social ties) [18], [19], we pose the following question: *Can we overcome the deficiency of individual records by taking advantage of their associated social ties*? The core intuition behind this idea is that even if a user possesses only a small number of check-in records, we can link the individual to their corresponding group of social ties, from which they may borrow additional or even substantial data and features.

So we first analyze the general amount of social ties for each user in Foursquare-Twitter datasets (c.f. Fig. 2(a)). Our findings indicate that more than half of the users have less than five social ties, which are inadequate to sufficiently support mining user features. This scarcity of social ties poses a challenge in characterizing users effectively. Next, we delve into the investigation of cross-domain common social ties for each user (see Fig. 2(b)). Notably, users with less than 50% common social ties account for a significant 80% of all users in the Foursquare dataset. Evidently, the presence of few common social ties is insufficient to enhance the cross-domain identity characterization, which necessitates the exploration and identification of additional common social ties for individuals.



(a) Number of predicted social links. (b) Proportion of predicted cross domain common social links.

Fig. 3. CCDF of the predicted number of social links and the proportion of common social links in Foursquare and Twitter datasets.

Upon initial consideration, the task seems analogous to link prediction, which aims to reveal missing or potential connections. Consequently, we assess the feasibility of using three typical link prediction methods on Foursquare-Twitter datasets for the UIL task, including persona2vec (social ties only) [20], walk2friends (mobility only) [21] and Vec2Link (both social ties and mobility) [10]. We present the CCDF of the number of predicted social links and common social links in Fig. 3. Upon analyzing the results, we observe that, compared to the original social ties, the number of predicted social links increases to some extent. However, all three algorithms fail to enhance the proportion of predicted common social links, and the predicted cross-domain social links of the same person exhibit little overlap. Even the best-performing algorithm Vec2Link only shows that 18% of users have more than 50% overlap, indicating that almost half of the predicted cross-domain social links are inconsistent for most users. As a consequence, the predicted discrepant social links hold little meaning for user feature augmentation in the UIL task, and none of the existing link prediction methods can adequately satisfy the demand for mining common social links.

C. Motivation

There are two major reasons behind the under-performance of existing link prediction studies. First, current works focus on predicting social links solely utilizing the social ties within a single platform, while our intention is to explore the potential cross-domain social links. In reality, users are likely to follow different accounts in different domains, where direct contact may not occur. Consequently, employing social ties indiscriminately results in little overlap among the predicted social links. Second, most mobility-based link prediction methods concentrate on predicting a user's mobility neighbors by modeling their visiting patterns. However, with insufficient check-in records, it becomes inaccurate to model user mobility behavior using probabilistic models. Furthermore, a group of users with similar

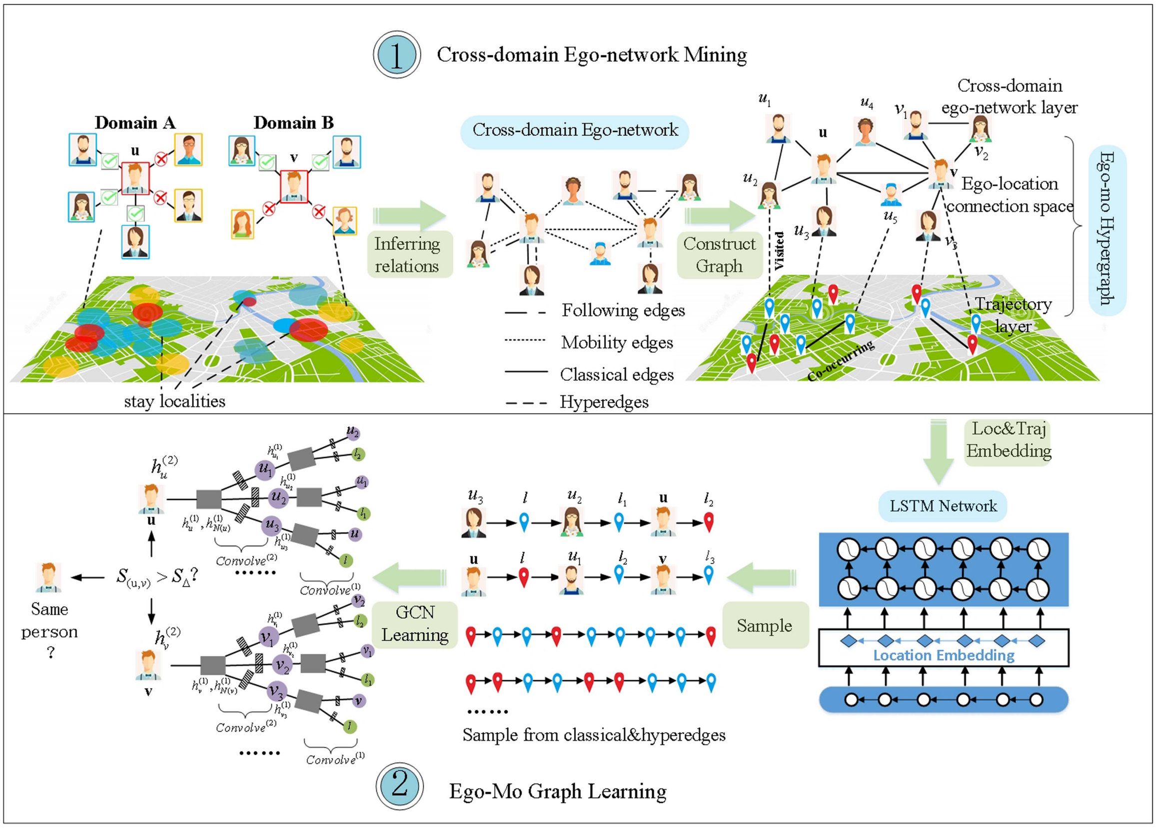


Fig. 4. Pipeline of EgoMUIL. Given target users u and v from social networks A and B, a mining function is performed to construct a cross-domain ego-network by filtering potential close friends from followings and mobility relations. A GCN is applied for learning user representation aggregating information in ego-mo hypergraph and calculate the similarity for users.

frequently visited spots may not all be considered as close friends due to the possibility of contingency.

These reasons emphasize the necessity of cross-domain relationship mining, combining available social ties and mobility overlap. This motivation drives us to leverage spatio-temporal information from socially closely-related users to facilitate improved user representation learning and linkage. To achieve social tie correlation, our approach involves assigning each follower-followee with a specific closeness metric using topology similarity, which helps distinguish genuine social ties from accidental connections. For mobility correlation, we propose to extract stay localities for each user to stand for the mobility feature and determine the closeness according to the overlap, which can relieve the sparsity problem.

Hence, for a given user, we refer to users with a certain mobility overlap as mobility neighbor, and those with social tie topological connection as social neighbor. Nevertheless, we impose mobility overlap restriction on the social tie topological connection. Social neighbors that coincide with mobility neighbors or have compact social tie topological connections are considered closely-related social links.

III. EgoMUIL Design

For two different LBSNs with corresponding two sets of user accounts $U: u_1, u_2, \ldots, u_m$ and $V: v_1, v_2, \ldots, v_n$, EgoMUIL aims to find all cross-domain linked common user pairs $(u_i, v_j), i \in (1, \ldots, m), j \in (1, \ldots, n)$, given limited check-in records of each user and the respective social relationships E_U and E_V, where E_U and E_V represent the following edges between the users. To achieve this, EgoMUIL involves the following two main steps (c.f. Fig. 4):

1) Cross-domain ego-network mining: This step synthetically utilizes topology structures from follow relationships and locality area similarity. It constructs a weighted cross-domain ego-network that screens out closely-related users.

2) Ego-mo hypergraph learning and linkage: This step extends the single-layer cross-domain ego-network to an ego-mo hypergraph. It utilizes GCN to aggregate neighbor features and output linked user pairs by calculating the similarity. We summarize the meaning of notations in Table II.

A. Cross-Domain Ego-Network Mining

To exploit the closely-related social links, we define a novel local community structure as the cross domain ego-network, which is based on the node-centric structure called the ego-network [22]. It is a subgraph that only contains the social ties of a specific node:

Definition 1 (Ego-network [22]): The ego-network of a user $u \in U$ is a simple microscopic social network model consisting

TABLE II
NOTATIONS AND CORRESPONDING MEANINGS

Notation	Meaning
U, V	The set of user accounts of LBSNs
u_i	The i-th user account in U
v_j	The j-th user account in V
E_U	The following edges between users in U
E_V	The following edges between users in V
$\mathcal{N}_u^f$	The set of following neighbors of user u
$\mathcal{E}_u^f$	The social edges between user u and its followees
$\mathcal{E}_u^m$	The mobility edges of user u from stay locatity
C_u	The center set for user u
$e_{u,u}$	The edges between different users
$e_{l,l}$	The edges between different locations
$e_{u,l}$	The edges between a user and a location
Φ_{l_1}	The location embedding
T_s	The sub-trajectory with location embeddings
H	The user embedding

of u (called ego) and all the following neighbors $\mathcal{N}_u^f$ with whom the ego has direct social ties $\mathcal{E}_u^f$, i.e., $\mathcal{G}_u^f : \{u, \mathcal{N}_u^f, \mathcal{E}_u^f\}$.

The ego-network is confined to an isolated platform where the social ties are limited to mere online follower-followee relationships. However, as mentioned in Section II, users tend to follow different people in different networks, indicating that follow relations alone may not be sufficient for relation mining. Inspired by mobility-based link prediction studies [23], [24], [25], we propose to explore additional cross-domain offline stay locality connections to establish closely-related social links and construct the cross-domain ego-network.

Definition 2 (Cross-domain Ego-network): The cross-domain ego-network of a user $u \in U$ is a microscopic local community where u is surrounded by a set of cross domain neighbors $\mathcal{N}_u$ with connected links $\mathcal{E}_u$, i.e., $\mathcal{G}_u = \{u, \mathcal{N}_u, \mathcal{E}_u\}$. Specially, the links $\mathcal{E}_u$ are integrated by following edges $\mathcal{E}_u^f$ from online social ties and mobility edges $\mathcal{E}_u^m$ from offline stay locality connections.

The basic idea here is that, the following/mobility relation which is confirmed to have high overlap of following/mobility association is considered as the highly-related relation. Therefore, we design the topology similarity based on common neighbors to characterize the follow relations, and utilize a density-aware hierarchical clustering method to define the stay locality similarity to reveal the mobility association. Still, following neighbors which have high degree of closeness or are mobility neighbors at the same time can be considered into cross-domain ego-network, which is similar for mobility neighbors. In more detail, following neighbors that have a high degree of closeness or are mobility neighbors at the same time can be considered into the cross-domain ego-network. The same principle applies to mobility neighbors. This approach ensures that the cross-domain ego-network incorporates users with significant commonality in their social ties and mobility patterns, allowing EgoMUIL to capture meaningful and closely-related social links between users in different LBSNs.

Topology Similarity: It is revealed that users with more common neighbors tend to have a tighter connection [26]. Also, it is confirmed in a mature Adamic/Adar score research [27] that the common neighbor $x \in \mathcal{N}_{u_1} \cap \mathcal{N}_{u_2}^f$ with fewer neighbors contributes more to the relationship between u_1 and u_2. Based on these findings, we leverage the attribute of common neighbors to define topology similarity as follows:

$$Sim_t(u_1, u_2) = \sum_{u_x \in \mathcal{N}_{u_1}^f \cap \mathcal{N}_{u_2}^f} \frac{|\mathcal{N}_{u_1}^f \cap \mathcal{N}_{u_x}^f| \cdot |\mathcal{N}_{u_2}^f \cap \mathcal{N}_{u_x}^f|}{|\mathcal{N}_{u_x}^f|^2}, \quad (1)$$

where u_x is one of the common neighbors of u_1 and u_2, $|\cdot|$ is the cardinality of the set, and $|\mathcal{N}_x^f|^2$ is used to reduce the weight of the effect of common neighbors who have too many friends. This formulation allows us to capture the significance of common neighbors in determining the relationship strength between two users, emphasizing the importance of common neighbors with fewer connections, which is more indicative of a strong relationship.

Stay Locality Similarity: To model the mobility characteristics of each user, we propose a method to cluster the visited locations into stay localities based on their visiting frequency and density. The idea is to identify users with high overlap of stay localities, as they are likely to be associated users, referred to as mobility neighbors. Our approach is different from most existing works that measure the number of common visited places or model the probability of users accessing certain locations using probabilistic models. Instead, our method can effectively address the effect of sparse locations and better describe a user's scope of activities.

The key to discovering stay localities is to determine the clustering centers, which are expected to be the most frequently visited locations with the densest surroundings. To achieve this, we define the centers based on the access preference score, which is determined by both the access frequency and the relative density of the locations. For a user u with a location set $\mathcal{L}u$, the access preference score $\mathbb{P}_{l_u^i}$ of location $l_u^i \in \mathcal{L}_u$ is calculated as

$$\mathbb{P}_{l_u^i} = \rho_{l_u^i} \times f_{l_u^i}, \quad (2)$$

where $f_{l_u^i}$ is the visited frequency to l_u^i and $\rho_{l_u^i}$ is the relative density, defined as its Radial Basis Function approximation to all other locations in $\mathcal{L}_u$. It reflects $\rho_{l_u^i}$'s local density among all locations in $\mathcal{L}_u$ and is computed as

$$\rho_{l_u^i} = \sum_{l_u^j \in \mathcal{L}_u \setminus \{l_u^i\}} \exp(-|l_u^i - l_u^j|^2). \quad (3)$$

We select candidate centers $\mathcal{L}_{can,u}$ with the top-K highest frequency relative density values and decide the final centers based on the minimum distance criterion. For each candidate center $l_{can,u}^i \in \mathcal{L}_{can,u}$, its minimum distance $d_{l_{can,u}^i}$ to other candidate centers is calculated as

$$d_{l_{can,u}^i} = \min_{l_{can,u}^j \in \mathcal{L}_{can,u} \setminus \{l_{can,u}^i\}} |l_{can,u}^i - l_{can,u}^j|. \quad (4)$$

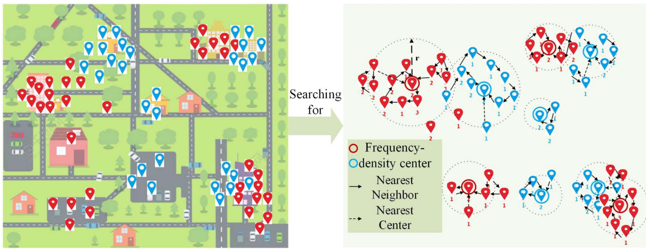


Fig. 5. Process of density-aware hierarchical clustering. Starting from the center of each user, we initially identify its closest location (indicated by the black arrow) until reaching a closed loop. Subsequently, we allocate the locations along the path to the nearest center (indicated by the black dotted arrow).

A distance threshold d_s is utilized to determine the final center with $d_{l^k_{can,u}} > d_s$. After careful consideration and experimentation, we set $d_s = 2$ km as displayed in Table III.

After obtaining the center set C_u for user u, we use a hypothesis inspired by the concept of reciprocal nearest data points [28] to assign the remaining locations to their nearest clusters. Specifically, starting from the centers, each location is required to find its conditional nearest location (within the distance of 1 km) to form a path until it reaches a closed loop. Locations without conditional nearest neighbors are excluded as noise. Locations along the path are considered as part of a cluster. The illustration of this process is shown in Fig. 5.

Based on this hypothesis, points in the clusters tend to form a uniform approximation of a circle rather than a narrow rectangle, due to the shortest path and conditional nearest location principle. Each clustering is abstracted into a geo-coverage disk $disk^i_u$ with its center c^i_u and the radius

$$r^i_u = \sqrt{\frac{1}{n}\sum_{j=1}^{n} |l^j_u - c^i_u|}, \tag{5}$$

where n is the number of locations in the clustering centered on c^i_u. For clusters with only one location, we set the default radius $r = 1$ km. We define each clustering as a stay locality of the user, which is abstracted into a geo-coverage disk for ease of calculation. Then, the stay locality similarity between users u_1 and u_2 can be calculated as the overlapping degree of their geo-coverage disks

$$Sim_l(u_1, u_2) = \sum_{i}^{a}\sum_{j}^{b} \frac{S\{disk^i_{u_1} \cap disk^j_{u_2}\}}{S\{disk^i_{u_1} \cup disk^j_{u_2}\}}, \tag{6}$$

where a and b are the number of stay localities of u_1 and u_2, and $S\{disk^i_{u_1} \cap disk^j_{u_2}\}$ and $S\{disk^i_{u_1} \cup disk^j_{u_2}\}$ are the areas of intersection and union of their stay localities. Users with higher stay locality similarity have a higher overlapping activity range.

With the topology similarity and stay locality similarity, we define that user u_2 is part of u_1's cross-domain ego-network if their topology similarity and stay locality similarity are both higher than the specific thresholds σ_1 and σ_2 respectively. In some cases, users may have a strong preference for a specific platform, so we also consider users with particularly high single similarity, meaning that if two users have extremely similar circles of friends, they play an important role in characterizing each other's social circles. The thresholds σ_1 and σ_2 are chosen based on careful consideration and are displayed in Table III. Using this method, we can identify all cross-domain ego-networks for all users.

B. Ego-Mo Hypergraph Learning

After obtaining the cross-domain ego-network for each user, we further concentrate on identifying cross domain user pairs with the help of additional spatio-temporal information. It requires us modeling the spatio-temporal information and the social links at the same time to support information aggregation. Therefore, we come up with a novel ego-mo hypergraph which contains both the user-user relation and the check-in records.

To accomplish a clearer structure where users can benefit from adjacent users and locations, we divide the graph into two layers, which is more formally defined as:

Definition 3 (Ego-Mo Hypergraph): The Ego-mo hypergraph consists of two layers: the cross-domain ego-network layer and the trajectory layer. These two layers are connected by user access location incidents, which we refer to as ego-location connections, and the locations themselves are connected by co-occurrences. We define edges $e_{u,u}$ and $e_{l,l}$ between homogeneous nodes as classical edges. In the ego-location connection space, edge $e_{u,l}$ represents a hyperedge that signifies the connection between a user and a location.

In the ego-mo hypergraph, each location is connected to others, regardless of the resource (i.e., which social network it belongs to), and this design is conducive to finding location dependencies and transition patterns. Accordingly, each user can be represented as vectors through location and social link aggregation using GCN. Finally, users with adjacent vectors can be considered to be the same person, facilitating the identification of cross-domain linked common user pairs.

Ego-Mo Hypergraph Construction: To portray users' mobility characteristics along with their neighbors for better information learning, we construct an ego-mo hypergraph by merging various types of information, including relations between user-user, user-location and location-location.

For candidate cross-domain nodes u and v, their ego-companies $\mathcal{G}_u$ and $\mathcal{G}_v$ are distributed in the ego-companies layer. As calculated in Section III-A, for example, if u_1 is a neighbor of u, their similarity is defined as

$$Sim(u, u_1) = Sim_t(u, u_1) + Sim_l(u, u_1). \tag{7}$$

To sample homogeneous nodes, after obtaining similarities of all neighbors, we normalize them using a softmax function to assign edges with comparable weights

$$\alpha_{uu}(u, u^i) = \frac{\exp(Sim(u, u^i))}{\sum_{u^j \in \mathcal{G}_u} \exp(Sim(u, u^j))}. \tag{8}$$

In the ego-location connection space, events of users visiting specific spots are represented. Since we are sampling homogeneous and heterogeneous edges separately, weights of user-location edges, e.g., α_{ul}, are directly given by the visiting frequency rather than participating in normalization.

The trajectories layer consists of access locations of all users where the weights are provided by the co-occurring frequency between locations. To achieve this, without loss of generality, we first divide the whole trajectory of user u into segments according to a time interval Δt, meaning locations in a common sub-trajectory are considered to be co-occurring. Similarly, we assign normalized weights α_{ll} to edges between locations based on the co-occurring frequency.

User and Location Representation Learning: As mentioned earlier, both users and locations are treated as nodes capable of gathering information from their surrounding neighbors. Therefore, both need to be embedded into vectors to provide valid node representations for the following GCN. We define the generated trajectory as the user node embedding, which captures the location node embedding from all the trajectories.

For the location embeddings, we draw inspiration from word embeddings, as there is a similarity between location frequency and word frequency, both of which follow a power-law distribution [29]. To learn location embeddings, we mix the sub-trajectories of all users indiscriminately and learn from the context. Specifically, for each location l_i, given its context locations $Ctx(l_i, l) = \{l_{i-m} : l_{i+m}\}$, we learn its embedding $\Phi(l_i)$ by maximizing the likelihood of $p(\Phi(l_i)|Ctx(l_i, l))$ which is defined as

$$\begin{aligned} &p(\Phi(l_i)|Ctx(l_i, l)) \\ &= \prod_{l' \in Ctx(l_i,l)} \frac{\exp\{\Phi(l_i) \cdot \Phi(l')\}}{\sum_{l'' \in Ctx(l_i,l)} \exp\{\Phi(l'')\Phi(l')\}}. \end{aligned} \tag{9}$$

When learning the user embeddings, since treating trajectories as a set of locations would loss the temporal information of this user's mobility pattern, we first separate the whole trajectory into multiple sub-trajectories and leverage these sub-trajectories to construct the user embedding. Then we feed the sub-trajectories into a network to obtain sequential vector representations. Since Long Short-Term Memory (LSTM) [30] is able to capture the temporal patterns and dependencies, we naturally select LSTM to obtain the user embedding with preserving the spatio-temporal information. For an input sub-trajectory with location embeddings $Ts : \Phi_{l_1}, \Phi_{l_2}, \ldots, \Phi_{l_T}$, the LSTM network calculates the unit activations continuously from $t = 1$ to T.

The LSTM updates its output h_t based on the current input Φ_{l_t} and the previous hidden state h_{t-1}, while a memory cell C_t is used to record the current state. The LSTM includes forget, input, and output gates, which selectively filter and retain information, allowing it to capture long-term dependencies in sequences. The current memory state C_t is updated by selectively inheriting the previous C_{t-1} based on the output f_t of the forget gate. Additionally, a candidate cell state $\widetilde{C_t}$ is generated using the $tanh$ activation function, and an input gate i_t is used to control the flow of information. The input gate and candidate cell state are calculated as follows:

$$i_t = \sigma(W_{ix}\Phi_{l_i} + W_{im}h_{t-1} + W_{ic}C_{t-1} + b_i) \tag{10}$$

$$\widetilde{C_t} = tanh(W_{cx}\Phi_{l_i} + W_{cm}h_{t-1} + b_c). \tag{11}$$

The current memory cell state C_t is then updated as

$$C_t = f_t \odot C_{t-1} + i_t \odot \widetilde{C_t}. \tag{12}$$

The output gate o_t is used to pass part of the memory cell C_t to the remainder of the network, and is calculated as

$$o_t = \sigma(W_{ox}\Phi_{l_i} + W_{om}h_{t-1} + W_{oc}C_{t-1} + b_o). \tag{13}$$

Finally, the current hidden state h_t is determined by the output gate with the current memory cell state C_t, using the $tanh$ function to limit the value between 0 and 1

$$h_t = o_t \odot \tanh(C_t). \tag{14}$$

The trajectory embeddings are updated by updating the hidden state of the LSTM, which is effective in handling long trajectory sequences. Thus, we obtain users' trajectory embeddings through the final hidden state h_t.

Node Representation Learning from GCN: GCN allows us to extract rich information by propagating edge information to aggregate neighbor node features. In our case, we use a two-layer GCN to focus on the variable range of the ego-mo hypergraph, where relations between user-user, user-location and location-location are propagated. The propagation rule of the method is

$$h_i^{(l+1)} = \sigma\left[\sum_{r \in \mathcal{R}} \widehat{D}_r^{-0.5} \widehat{A}_r \widehat{D}_r^{-0.5} h_i^{(l)} \alpha_r\right]. \tag{15}$$

This formula is derived from the Graph Fourier Transformation where Symmetric normalized Laplacian [31] is applied, which is currently utilized in graph theory as a matrix representation of a graph, and is calculated as

$$L^{sys} = \widehat{D}_r^{-0.5} L \widehat{D}_r^{-0.5}, \tag{16}$$

where L is the Laplacian matrix and the elements of the matrix is

$$[L_{i,j}] = \begin{cases} 1, & i = j \text{ and } d(v_i) \neq 0, \\ \frac{1}{\sqrt{d_{v_i} d_{v_j}}}, & i \neq j \text{ and } v_i \text{ is adjacent to } v_j, \\ 0, & \text{otherwise} \end{cases}, \tag{17}$$

where d_{v_i} is the degree of vertex v_i that represents the number of linking edges.

In particular, we represent each node as a vertex in the graph, and the corresponding edges are denoted by $r \in \mathcal{R} = uu, ul, ll$, which capture relations between user-user, user-location, and location-location. In the GCN formula, $h_i^{(l)}$ represents the hidden layer of vertex V_i in layer l, and h_i^1 corresponds to the input layer. The matrix $\widehat{D_r}$ measures the corresponding degree matrix, which is a diagonal matrix in the form of

$$\begin{pmatrix} d_{v_{i_1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_{v_{i_n}} \end{pmatrix}, \tag{18}$$

where the elements d_{v_i} on the diagonal are the degrees of each vertex that represent the number of edges associated. Moreover, α_r is the weight of each relation edge obtained when constructing the ego-mo hypergraph. α_{uu} is calculated by the topology and stay locality similarity with a softmax function normalization, α_{ul} is given by the user visiting frequency and α_{ll} is defined by the co-occurring frequency. Besides, for ego-location connection space $r = ul$, $\widehat{A}_r = A_r$ where A_r is the adjacency matrix storing the adjacency information between vertices. For cross-domain ego-network layer and trajectory layer $r \in \{uu, ll\}$, $\widehat{A}_r = A_r + I$ which adds self-connections when aggregating adjacencies.

Since the users within an ego-network usually have a high closeness, and the users in different ego-network usually separate far away with each other, the users within the same ego-network should have similar representations. In order to encourage similar nodes to be adjacent in the embedding space, we design a loss function for each node u, where we sample nodes in its local context. The loss function is defined as follows:

$$\mathcal{L}(u) = -\sum_{n_i \in \mathcal{C}tx_u, r \in \mathcal{R}} \lambda_r \log(H_u H_{n_i}). \tag{19}$$

Here, λ_r represents the given weights for the three types of relations: user-user, user-location, and location-location. The set $\mathcal{C}tx_u$ contains the sample nodes of node u, obtained through a combination of breadth-first sampling and depth-first sampling from node2vec. Node u can represent either a user or a location node.

The objective is to minimize this loss function. Ultimately, the final loss is computed as the average of all the nodes, including users in U, users in V, and all their access locations in Loc

$$\mathcal{L}oss = \frac{\sum_{u \in \{U,V,Loc\}} \mathcal{L}(u)}{|U| + |V| + |Loc|}. \tag{20}$$

By optimizing the aforementioned loss function, users in close proximity will exhibit similar embeddings across different LBSNs.

To summarize, the process of learning the final representations for users from the ego-mo hypergraph involves obtaining the trajectory and location embeddings using LSTM. These embeddings are then fed into the two-layer GCNs, where local information is aggregated and forwarded through the layers. The final embeddings are trained by minimizing the loss function described in (20). As a result, we obtain the vector representations H_u for all users across networks, capturing the comprehensive information from user-user, user-location, and location-location relations.

C. User Identity Linkage Across Networks

After obtaining the vectors H_u and H_v for the focal pair (u, v) representing the cross-domain nodes, we need to determine whether the two nodes belong to the same user. To achieve this, we utilize vector similarity measures such as Cosine similarity and Pearson correlation coefficient (PCC), which are denoted as

$$\mathrm{Cos}(H_u, H_v) = \frac{< H_u, H_v >}{||H_u|| \cdot ||H_v||} \tag{21}$$

$$Corr(H_u, H_v) = \frac{< H_u - \overline{H_u}, H_v - \overline{H_v} >}{||H_u - \overline{H_u}|| \cdot ||H_v - \overline{H_v}||}, \tag{22}$$

where Cosine similarity principally measures the angle between two vectors and PCC takes the translation invariance into account and realize the dimension correlation.

In our approach, we conduct both measurements on the final output embeddings, and the results are visualized in Fig. 9, where Cosine similarity fits our approach better. Therefore, the final similarity between user embeddings is decided using Cosine similarity

$$S_{u,v} = \mathrm{Cos}(H_u, H_v). \tag{23}$$

Finally, given two sets of user accounts $U: u_1, u_2, \ldots, u_m$ and $V: v_1, v_2, \ldots, v_n$ from two social network platforms, we calculate the Cosine similarity for each user pair. A threshold S_Δ is used to decide whether two nodes belong to the same person: if $S_{u,v} \geq S_\Delta$, they are considered to belong to the same user. We implement the linkage under varying threshold S_Δ and determine an optimal value $S_\Delta = 0.95$. This process helps us identify the cross-domain linked common user pairs efficiently and accurately.

IV. Performance Evaluation

A. Experiment Setup

1) Datasets: We use real-world check-in datasets from Foursquare and Twitter in our experiments, which are provided by [14], and are commonly adopted by recent works [5], [8], [15], [16], [32], [33], [34]. We extract the spatio-temporal information from the check-in records and retain the follow relationship among users. Data statistics are shown in Table I in Section II, with 1,644 linked ground truth user pairs.

2) Compared Algorithms: As EgoMUIL aims to generate user embeddings in LBSNs using spatio-temporal information, we compare our algorithm with the following methods for user embedding generation. It is important to note that EgoMUIL distinguishes itself by integrating data from different LBSNs to derive user embeddings, a characteristic not shared by the comparisons.

POIS [16]: This work exploits rare coincidences by making use of the available spatio-temporal records, and a Poisson process is leveraged to represent the sparse properties. The similarity between user pairs is calculated as

$$Sim_{u,v} = \sum_{t \in T} \sum_{l \in L} \phi_{l,t}(U(t)V(t)), \tag{24}$$

where $\phi_{l,t}$ measures the probability of an "encountering" event in location l at time slot t.

SIMP [15]: This method presents a contact graph model for multi-service ID linkage where all the IDs are mapped into the graph and the co-located events are considered as edges between nodes. It employs Bayesian based optimal ID matching algorithm to identify the most probable ID sets where the similarity between user pairs is

$$Sim_{u,v} = P(X(u,v)=1|r(V)) = \frac{Q(u,v)}{\sum_{u\in N^{s_1}(v)} Q(u,v)+\beta(v)r(V)}, \quad (25)$$

where $N^{s_1}(v)$ represents a collection of ID accounts in V, $Q(u,v)$ is defined as the joint probability of the observation $r(V)$ and $X(u,v)=1$, and $\beta(v)$ is the probability that v does not in $N^{s_1}(v)$.

GKR-KDE [8]: This work proposes a grid-based structure to organize locations. A kernel density estimation (KDE) based method is then utilized to characterize individual spatial patterns, where the similarity between user pairs is

$$Sim_{u,v} = \sum_{i=1}^{k} f(g_{u_i}|G(v),h), \quad (26)$$

where $G(v)$ is the grid representation of user v; g_{u_i} is the grid cell ID of user u; $f(g_{u_i}|G(v),h)$ is the probability density function, which measures the distance between g_{u_i} and $G(v)$, and its radial range is controlled by a bandwidth parameter h.

DPLink [4]: This method extracts and compares representative features from trajectories, and a co-attention mechanism is then designed to cope with the low quality of mobility data. The linkage is considered as a binary classification problem and the binary cross entropy loss is employed as the objective loss function.

DeepLink [35]: This approach encodes the users into a vector representation in a semi-supervised learning manner, where only the network structure is required. A mapping function Φ is learned by minimizing the following loss function:

$$Loss_{e(u_i),e(v_j)} = \min(1-\cos(\Phi(e(u_i),e(v_j)))), \quad (27)$$

where $e(u_i)$ and $e(v_j)$ are the vector representation of the cross domain users u_i and v_j, respectively.

Furthermore, to assess the impact of the cross-domain ego-network mining module and the ego-mo hypergraph learning and linkage module on the performance of our EgoMUIL, we consider the following two baselines for the ablation study.

Baseline1: This baseline omits the utilization of the cross-domain ego-network mining module, in which we identify germane users to form cross-domain ego-networks. To assess the impact of this module, we directly designate the most closely-related user as the identified user, while keeping the remaining processes identical to those in EgoMUIL.

Baseline2: This baseline excludes the use of the ego-mo hypergraph learning and linkage module, where we gather and learn spatio-temporal information from neighbors to characterize individuals. In this baseline, we directly identify the follow relations as neighbors without constructing cross-domain ego-networks, while keeping the remaining processes the same as that in EgoMUIL.

TABLE III
TYPICAL PARAMETER SETTINGS

Item	Setting	Item	Setting
radius	$1\ km$	drop out	0.5
d_s	$2\ km$	epoch	$2,000$
Δt	$6\ h$	$\lambda_{uu}, \lambda_{ll}$	1
σ_1	$e^{1.1}$	λ_{ul}	0.9
σ_2	$e^{0.8}$	S_Δ	0.95

3) Metrics: We evaluate and compare the performance of existing methods though the following metrics:

Precision@k, which measures the accuracy of the linkage function and is calculated as

$$Precision@k = \sum_{i}^{n} \mathbb{1}success@k/n, \quad (28)$$

where $\mathbb{1}success@k$ measures whether the returned users include the correctly matched account.

MAP (Mean Average Precision), which is used to weigh the ranking performance of algorithms and is defined as

$$MAP = \left(\sum_{i}^{n} \frac{n_{rt}-n_{rk}}{n_{rt}}\right)/n, \quad (29)$$

where n_{rt} is the number of returned users, and n_{rk} is the rank of the correctly matched identity.

$\boldsymbol{Sen_r}$ and $\boldsymbol{Sen_f}$, which measure the probability of successful linked users with a small number of check-in records and social ties, respectively. They can reflect the sensitivity of the algorithm to the data scarcity ($Sen > 1$ indicating non-sensitive), and can be obtained by

$$Sen_r = \frac{P_{suc_r<\epsilon_r}}{P_{grt_r<\epsilon_r}} \quad (30)$$

$$Sen_f = \frac{P_{suc_f<\epsilon_f}}{P_{grt_f<\epsilon_f}}, \quad (31)$$

where ϵ_r and ϵ_f are the specific number of check-in records and social ties; suc_r and suc_f are the records and social ties number of correctly identified users; grt_r and grt_f are the records and social ties number of users in ground truth. $P_{suc_r<\epsilon_r}$ is the probability of users with less than ϵ_1 records correctly identified. We set $\epsilon_r = 5$ and $\epsilon_f = 5$ to evaluate the linkage performance with less than 5 records/social ties, unless otherwise specified.

4) Experiment Settings: We implement our EgoMUIL on Tensorflow platform and we carry out LSTM as the default recurrent network where both locations and trajectories are represented as 256-dimensional vectors, which are fed into a two-layer GCN with 128 units in the hidden layer. We utilize Cosine similarity to identify user pairs. Other parameters settings are summarized in Table III.

B. Performance of EgoMUIL

We classify the compared algorithms into traditional-based methods (including GKR-KDE [8], SIMP [15] and POIS [16]) and deep learning-based methods (including DeepLink [35],

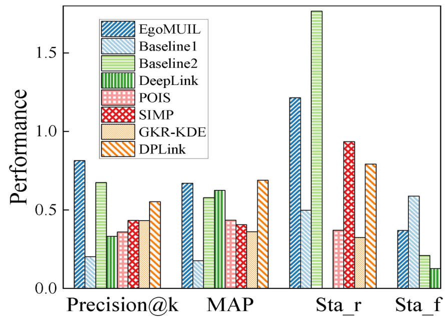


Fig. 6. Performance in Foursquare-Twitter.

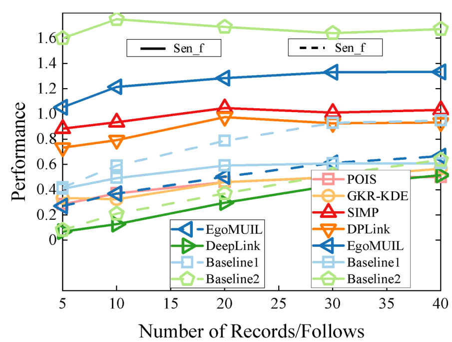
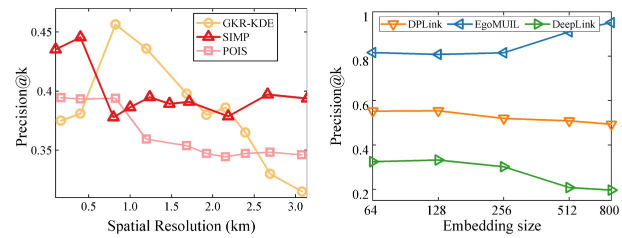


Fig. 7. Performance w.r.t. varied number of records/follows.



(a) Performance w.r.t. varied spatial resolution.

(b) Performance w.r.t. varied embedding size.

Fig. 8. Performance w.r.t. varied parameters: (a) Traditional methods; (b) Deep learning based methods.

DPLink [4] and EgoMUIL), on the basis of which we design different scenarios to test the performance.

We evaluate the performance of all methods in terms of accuracy ($Precision@k$), ranking performance (MAP), and sensitivity to scarcity (Sen_r, Sen_f), and the results are shown in Fig. 6. As observed, EgoMUIL stands out among all methods. Its exceptional performance can be attributed to its special design, taking into account the scarce property of both check-in records and followings. EgoMUIL demonstrates remarkable sensitivity to scarcity, outperforming other methods in this aspect. To further verify the robustness and superiority of EgoMUIL when faced with few records and follows, we examine Sen_r and Sen_f of all methods, specifically focusing on the success of identified Twitter records/follows in Fig. 7.

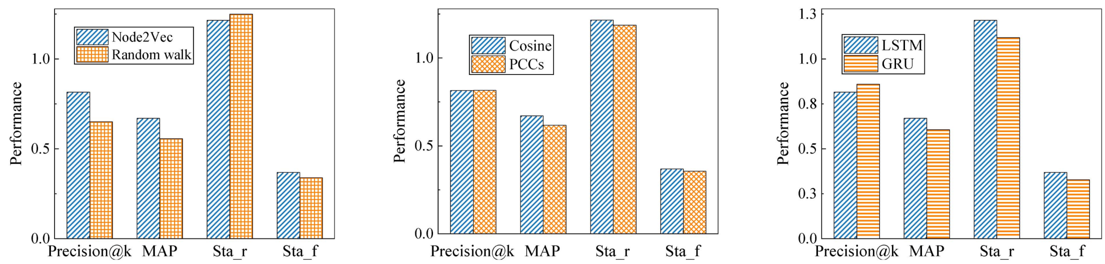
Furthermore, traditional-based methods decide trajectory similarity by aligning available locations, which are coarse-grained through bins or grids. As a consequence, these methods are sensitive to spatial resolution. To investigate this sensitivity, we apply these algorithms at varying spatial resolutions and analyze the results, as shown in Fig. 8(a). We find that the grid-based GKR-KDE is most affected, and its performance decreases significantly with the increase of spatial resolution. In addition, we test the performance of deep learning-based methods under various embedding sizes, as illustrated in Fig. 8(b). The results show that DPLink and DeepLink perform best with an embedding size of 128, and their performance declines slightly as the embedding size increases. This indicates that a small number of dimensions is sufficient for these algorithms. On the other hand, our algorithm, EgoMUIL, performs better as the embedding size increases due to its capability of aggregating ample information from neighbors.

C. Impact of Different Settings

First, when learning the representation of nodes, we obtain local context samples of the user-user, user-location, and location-location relations using Node2Vec based on the constructed cross-domain ego-network. We consider the frequency of user visits and the number of simultaneous occurrences in a trajectory when generating these samples. To validate the advantage of this design, we compare our method's performance with a random walk-based approach, which samples the relations of nodes without considering weights. The results are visualized in Fig. 9(a). Our method outperforms the random walk-based approach in terms of accuracy and ranking, while achieving equivalent performance with respect to Sen_r and Sen_f. This indicates that our approach effectively utilizes the local context information and takes advantage of the weighted samples, leading to improved performance in user representation learning and linkage. The results further demonstrate the superiority of EgoMUIL in the UIL task, highlighting its capability to leverage valuable information from the constructed cross-domain ego-network.

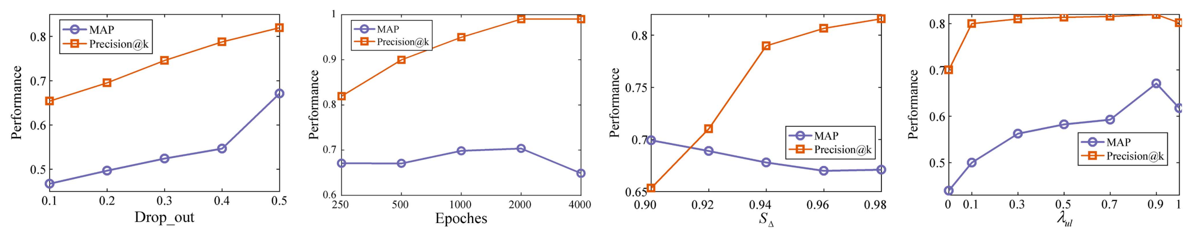
Second, to evaluate the utility of the two modules in our EgoMUIL, we compare the performance of EgoMUIL with aforementioned two baselines (c.f. Section IV-A2) on the same dataset. The results of these separate evaluations are shown in Fig. 9(b). The results indicate that both baseline1 and baseline2 underperform in terms of ranking and accuracy compared to the complete EgoMUIL approach. Still, they also demonstrate advantages in different aspects, where baseline1 outperforms in scenarios with few check-in records, and baseline2 is less affected by scarce follow relations. These benefits can be attributed to the superior design of the respective modules.

Third, when calculating the final similarity and identifying users, we utilize the widely used Cosine similarity. Additionally, we consider using PCC as an alternative method to measure the correlation coefficient of two vectors for similarity calculation. To compare the performance of Cosine and PCC, we present the results in Fig. 9(c). The comparison shows that both Cosine and PCC measurements yield corresponding performance, with Cosine slightly outperforming PCC. Although both similarity metrics are suitable for vector similarity calculation, Cosine



(a) Node2Vec v.s. Random Walk. (b) Cosine v.s. PCCs. (c) LSTM v.s. GRU.

Fig. 9. Performance under different settings in EgoUIL.



(a) Performance w.r.t. varied Drop_out. (b) Performance w.r.t. varied Epoches. (c) Performance w.r.t. varied λ_{ul}. (d) Performance w.r.t. varied S_Δ.

Fig. 10. Performance w.r.t. different parameters in EgoUIL.

demonstrates a slightly better performance in the context of our EgoMUIL framework.

Finally, when learning sequential vector representations of sub-trajectories, we use LSTM to complete the task. Another representative recurrent neural network, the Gate Recurrent Unit (GRU), features only one gate, which requires fewer parameters compared to LSTM. To compare their effects, we replace LSTM with GRU in EgoMUIL and examine the linkage results, as shown in Fig. 9(d). While GRU is computationally efficient due to its reduced number of parameters, LSTM demonstrates better performance in learning sequential vector representations of sub-trajectories and achieving superior linkage results in the UIL task.

D. Impact of Different Parameters

Complementary to different settings, we also test the performance under varying training parameters in GCN: training times $epoch$, dropout and the weight of user-location relations λ_{ul} in (19). Specifically, in EgoMUIL, we set $\lambda_{uu} = 1$ and $\lambda_{ll} = 1$ directly. Additionally, the returned user pairs are largely dictated by the similarity threshold S_Δ in (23). We evaluate the impact of varying S_Δ and training parameters in Fig. 10, and the results indicate that:

- EgoMUIL can achieve a satisfactory performance within a certain range of drop out.
- With varying training times $epoch$, EgoMUIL converges to its best result when $epoch$ reaches about 2,000 times. Beyond this point, it may incur overfitting problems. Hence, an appropriate number of training times $epoch$ is sufficient.
- When sampling the heterogeneous relations from the ego-mo graph, the weight λ_{ul} greatly affects the ranking performance. EgoMUIL achieves its best performance with $\lambda_{ul} = 0.9$, where valuable heterogeneous nodes are adequately sampled.
- The threshold S_Δ significantly affects the ranking and accuracy performance. We set a compromise value of $S_\Delta = 0.95$ as the final choice.

In summary, the performance of EgoMUIL is influenced by various training parameters, and finding the right balance is crucial for achieving better results.

V. Related Work

A. Spatio-Temporal UIL

We classify spatio-temporal UIL algorithms into two categories: traditional methods and embedding-based methods.

Traditional methods typically utilize location-based similarity measurements by aligning users' trajectories and transforming locations into regions using bins or grids. These approaches often adopt the maximum weight mobility matching model [16], the location-based contact graph [15], or the signal-based similarity [36] on each user pair to capture their contact. Along this line, researchers further introduce a locality-sensitive hashing based mechanism [13], [37] to reduce candidate pairs for matching, and a kernel density estimation based solution [38] is proposed to model individual-level location data aiming at alleviating the data sparsity problem. In general, these methods calculate the location similarity relying on the visiting frequency or approximate co-occurrence/co-location, which are inefficient when dealing with the location sparsity issue.

Embedding-based methods, on the other hand, focus on learning the transition relations of locations instead of handcraft mobility features. These methods recognize the significance of the order of locations appearing in a trajectory, which is generally overlooked by traditional methods. These methods try to make full use of the sequence feature of trajectories by embedding them into low dimensional vectors utilizing paragraph2vec [39] or recurrent neural network based deep learning algorithms [4]. Motivated by the the over-looked time-evolving issues in RNN-based approaches, an attention mechanism on graph embeddings [40] is introduced for time-evolving analysis, and a two-tier graph contextual embedding framework [41] is designed to capture the long-range dependencies. To circumvent asymmetric information across platforms, Shao et al. [42] propose to utilize text-location information with a 3D convolution neural network.

With the development of Graph Neural Network (GNN), many researchers propose the representation-based network alignment method to use GNN to generate node embeddings. Zhang et al. [43] leverage GNN to generate the embeddings of network nodes and then align different networks. With their alignment, they could align the common users in both Foursquare and Twitter. Zheng et al. [44] find that the similarities between unlabeled cross-network user pairs have significant impact on the accuracy of UIL. Therefore, they introduce the weakly supervised training manner into GNNs and propose JORA, which can learn self-adaptive similarities for unlabeled user pairs. Long et al. [45] propose DegUIL, which can learn high-quality node embeddings for the nodes with few neighbors. Then Tang et al. [6] propose a GNN-based encoder Adaptive Graph Attention Network, named AdaGAT, which can generate the user embedding with the user characters and following topology in the social networks. Recently, to tackle the issue of missing inference edges during training in link prediction scenarios, Tan et al. [46] propose a framework that can automatically, personally, and inductively identify optimal subgraphs for different edges when performing UIL.

On the whole, in spite of the pre-training designed for sparse trajectories, all these methods underperform with scarce mobility records.

B. Network-Based UIL

Focusing on the network structure of across domains, researchers also attempt to accomplish UIL through mining user follow relationships. In [47], Liu et al. learn the follower/followee representation for each user using network embedding method which can preserve the proximity of user associations, while in [35], [48] the local and global structures are captured by encoding the network into space in a semi-supervised learning manner. Recently, graph neural networks provide a new way for user representation learning and linkage. To alleviate the semantic gap on different platforms, a hybrid graph neural network is proposed [49] to unify the intra-user and inter-user representation learning. Chen et al. [50] devise multi-level graph convolutions, considering both local structure and hypergraph structure to prevent the cold-start issue instead of directly modeling the network structure itself. Jiao et al. [51] propose a hierarchical graph attention based network embedding framework to simultaneously perform link prediction and cross-domain linkage. Nevertheless, all these methods are confronted with limited follow relations and the diversity social structure across networks.

C. Link Prediction

Link prediction aims to infer the missing links from the obtained social network structure [20] or learn the potential associations from their mobility patterns [52]. To better exploit the latent link between users, Zhou et al. [10] present a novel approach that combines social relations and mobility preferences. They simultaneously employ a probabilistic factor model and a network embedding method to enhance link prediction performance. Yang et al. [53] propose a random-walk-with-stay scheme that preserves n-wise node proximity while learning check-ins from the constructed hypergraph. In [54], a real-time algorithm based on changes in user communities is introduced which can reveal the importance of community indicative characteristics. A more recent approach by Zhang et al. [55] focuses on capturing the continuity and sequentiality of trajectories. They propose a multiview matching network that learns location, time-series, and relation simultaneously. While these methods are enlightening within a single platform, they may not be directly suitable for cross-domain link prediction.

Compared with existing studies, our approach focuses on resolving the linkage accuracy defect with scarce mobility records. Inspired by the network-based methods, the follow relations are utilized to compensate for the limited individual trajectories. Additionally, instead of conducting link prediction solely based on an isolated network, we incorporate mobility similarity to renovate the relations. This enables us to reveal closely-related cross-domain common relations, which in turn facilitate user characteristic learning and linkage.

VI. Conclusion

In this paper, we have presented EgoMUIL, a novel graph representation learning method designed to address the data scarcity issue by effectively aggregating neighbor information. Particularly, we observe significant differences between user followings across networks, which cannot be directly considered as neighbors. As a solution, we propose a novel approach that repairs and extends the existing follow relationships by combining topology structure and stay locality similarity. This novel method results in a two-layer heterogeneous ego-mo-graph, where user-user, user-location, and location-location relations are clearly distributed. Based on the graph, we first learn the initial trajectory and location embedding vectors to stand for user nodes and location nodes, and then propose a two-layer GCN learning technique that can propagate information layer-wisely and learn the final embeddings for users. Finally, the output vectors is computed by the cosine similarity for identifying the matched nodes. We conduct extensive experiments, and the results indicate that our EgoMUL performs much better comparing with existing works in terms of both ranking and accuracy. In our future works, we aim to explore more intricate scenarios involving heterogeneous or incomplete user data across different LBSNs, with the goal of enhancing the robustness and effectiveness of our work.

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Haojun Huang received the BS degree in computer science from the Wuhan University of Technology, in 2005, and the PhD degree in communication and information engineering from the University of Electronic Science and Technology of China, in 2012. He is an associate professor with the School of Electronic Information and Communications, Huazhong University of Science and Technology, China. His current research interests include Internet of Things, network function virtualization, wireless communications, and artificial intelligence for networking.

Fengxiang Ding received the BE degree from Central South University, China, in 2019. She is currently working toward the MS degree in electronics and information engineering with the Huazhong University of Science and Technology, China. Her research interests focus on spatio-temporal data mining and social computing.

Hao Yin (Member, IEEE) received the BS, ME, and PhD degrees in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1996, 1999 and 2002, respectively. He is a professor with the Beijing National Research Center for Information Science and Technology, Tsinghua University, Beijing, China. His research interests span broad aspects of multimedia communications and computer networks. He was awarded the State Natural Science Award (Second Prize), in 2011 and the State Technological Invention Award (Second Prize), in 2012, and was appointed as a Yangtze River Scholar, in 2021.

Gaoyang Liu (Member, IEEE) received the BS and PhD degrees from the Huazhong University of Science and Technology, China, in 2015 and 2021, respectively. He is now a post-doctoral researcher with the School of Computing Science, Simon Fraser University, British Columbia, Canada. His research interests include machine learning, mobile sensing, and data privacy protection.

Chen Wang (Senior Member, IEEE) received the BS and PhD degrees from the Department of Automation, Wuhan University, China, in 2008 and 2013, respectively. From 2013 to 2017, he was a postdoctoral research fellow with the Networked and Communication Systems Research Lab, Huazhong University of Science and Technology, China. Thereafter, he joined the Faculty of Huazhong University of Science and Technology where he is currently an associate professor. His research interests are in the broad areas of wireless networking, Internet of Things, and mobile computing, with a recent focus on privacy issues in wireless and mobile systems. He is a senior member of ACM.

Dapeng Oliver Wu (Fellow, IEEE) received the BE degree in electrical engineering from the Huazhong University of Science and Technology, Wuhan, China, in 1990, and the PhD degree in electrical and computer engineering from Carnegie Mellon University, Pittsburgh, Pennsylvania, in 2003. He is currently a chair professor with the Department of Computer Science, City University of Hong Kong. His research interests are in the areas of networking, communications, signal processing, computer vision, machine learning, smart grid, and information and network security. He has served as an editor in chief of *IEEE Transactions on Network Science and Engineering*, associate editor for *IEEE Transactions on Cloud Computing*, *IEEE Transactions on Computers*, *IEEE Transactions on Wireless Communications* and *IEEE Transactions on Vehicular Technology*, and a guest-editor for *IEEE Journal on Selected Areas in Communications*.